Formulario de Cálculo Diferencial e Integral VER.4.3

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VALOR ABSOLUTO

$$|a| = \begin{cases} a & \text{si } a \ge 0 \\ -a & \text{si } a < 0 \end{cases}$$

$$|a| = |-a|$$

$$a \le |a| \text{ y } -a \le |a|$$

$$|a| \ge 0 \text{ y } |a| = 0 \Leftrightarrow a = 0$$

$$|ab| = |a||b| \text{ ó } \left|\prod_{k=1}^{n} a_k\right| = \prod_{k=1}^{n} |a_k|$$

$$|a+b| \le |a| + |b| \text{ ó } \left|\sum_{k=1}^{n} a_k\right| \le \sum_{k=1}^{n} |a_k|$$

EXPONENTES

$$a^p \cdot a^q = a^{p+q}$$

$$\frac{a^p}{a^q} = a^{p-q}$$

$$\left(a^p\right)^q = a^{pq}$$

$$(a \cdot b)^p = a^p \cdot b^p$$

$$\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$$

$$a^{p/q} = \sqrt[q]{a^p}$$

LOGARITMOS

$$\log_a N = x \Rightarrow a^x = N$$

$$\log_a MN = \log_a M + \log_a N$$

$$\log_a \frac{M}{N} = \log_a M - \log_a N$$

$$\log_a N^r = r \log_a N$$

$$\log_a N = \frac{\log_b N}{\log_b a} = \frac{\ln N}{\ln a}$$

$$\log_{10} N = \log N \text{ y } \log_e N = \ln N$$

ALGUNOS PRODUCTOS

$$a \cdot (c+d) = ac + ad$$

$$(a+b)\cdot(a-b) = a^2 - b^2$$

$$(a+b)\cdot(a+b) = (a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)\cdot(a-b) = (a-b)^2 = a^2 - 2ab + b^2$$

$$(x+b)\cdot(x+d) = x^2 + (b+d)x + bd$$

$$(ax+b)\cdot(cx+d) = acx^2 + (ad+bc)x + bd$$

$$(a+b)\cdot(c+d) = ac + ad + bc + bd$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$$

$$(a-b)\cdot\left(a^2 + ab + b^2\right) = a^3 - b^3$$

$$(a-b)\cdot\left(a^3 + a^2b + ab^2 + b^3\right) = a^4 - b^4$$

$$(a-b)\cdot\left(a^4 + a^3b + a^2b^2 + ab^3 + b^4\right) = a^5 - b^5$$

$$(a-b)\cdot\left(\sum_{k=1}^{n} a^{n-k} b^{k-1}\right) = a^n - b^n \quad \forall n \in \mathbb{N}$$

$$(a+b)\cdot\left(a^2 - ab + b^2\right) = a^3 + b^3$$

$$(a+b)\cdot\left(a^3 - a^2b + ab^2 - b^3\right) = a^4 - b^4$$

$$(a+b)\cdot\left(a^4 - a^3b + a^2b^2 - ab^3 + b^4\right) = a^5 + b^5$$

$$(a+b)\cdot\left(a^5 - a^4b + a^3b^2 - a^2b^3 + ab^4 - b^5\right) = a^6 - b^6$$

$$(a+b)\cdot\left(\sum_{k=1}^{n} (-1)^{k+1} a^{n-k} b^{k-1}\right) = a^n + b^n \quad \forall n \in \mathbb{N} \text{ impar}$$

$$(a+b)\cdot\left(\sum_{k=1}^{n} (-1)^{k+1} a^{n-k} b^{k-1}\right) = a^n - b^n \quad \forall n \in \mathbb{N} \text{ par}$$

SUMAS Y PRODUCTOS

$$a_1 + a_2 + \cdots + a_n = \sum_{k=1}^{n} a_k$$

$$\sum_{k=1}^{n} c = nc$$

$$\sum_{k=1}^{n} c a_k = c \sum_{k=1}^{n} a_k$$

$$\sum_{k=1}^{n} (a_k + b_k) = \sum_{k=1}^{n} a_k + \sum_{k=1}^{n} b_k$$

$$\sum_{k=1}^{n} (a_k - a_{k-1}) = a_n - a_0$$

$$\sum_{k=1}^{n} \left[a + (k-1)d\right] = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$= \frac{n}{2}(a + l)$$

$$\sum_{k=1}^{n} ar^{k-1} = a\frac{1-r^n}{1-r} = \frac{a - rl}{1-r}$$

$$\sum_{k=1}^{n} k = \frac{1}{2}\left(n^2 + n\right)$$

$$\sum_{k=1}^{n} k^2 = \frac{1}{6}\left(2n^3 + 3n^2 + n\right)$$

$$\sum_{k=1}^{n} k^3 = \frac{1}{4}\left(n^4 + 2n^3 + n^2\right)$$

$$\sum_{k=1}^{n} k^4 = \frac{1}{30}\left(6n^5 + 15n^4 + 10n^3 - n\right)$$

$$1 + 3 + 5 + \cdots + (2n-1) = n^2$$

$$n! = \prod_{k=1}^{n} k$$

$$\binom{n}{k} = \frac{n!}{(n-k)!\,k!}, \quad k \le n$$

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$$

$$(x_1 + x_2 + \cdots + x_k)^n = \sum \frac{n!}{n_1!\,n_2!\cdots n_k!} x_1^{n_1} \cdot x_2^{n_2} \cdots x_k^{n_k}$$

CONSTANTES

$\pi = 3.14159265359\ldots$

$e = 2.71828182846\ldots$

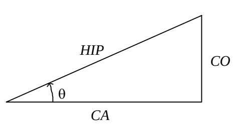
TRIGONOMETRÍA

$$\operatorname{sen}\theta = \frac{CO}{HIP} \qquad \csc\theta = \frac{1}{\operatorname{sen}\theta}$$

$$\cos\theta = \frac{CA}{HIP} \qquad \sec\theta = \frac{1}{\cos\theta}$$

$$\operatorname{tg}\theta = \frac{\operatorname{sen}\theta}{\cos\theta} = \frac{CO}{CA} \qquad \operatorname{ctg}\theta = \frac{1}{\operatorname{tg}\theta}$$

π radianes=180°



θ	sen	cos	tg	ctg	sec	csc
0°	0	1	0	∞	1	∞
30°	$1/2$	$\sqrt{3}/2$	$1/\sqrt{3}$	$\sqrt{3}$	$2/\sqrt{3}$	2
45°	$1/\sqrt{2}$	$1/\sqrt{2}$	1	1	$\sqrt{2}$	$\sqrt{2}$
60°	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$	$1/\sqrt{3}$	2	$2/\sqrt{3}$
90°	1	0	∞	0	∞	1

$$y = \angle\operatorname{sen} x \quad y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$y = \angle\cos x \quad y \in [0, \pi]$$

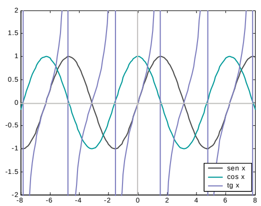
$$y = \angle\operatorname{tg} x \quad y \in \left\langle -\frac{\pi}{2}, \frac{\pi}{2}\right\rangle$$

$$y = \angle\operatorname{ctg} x = \angle\operatorname{tg}\frac{1}{x} \quad y \in \langle 0, \pi\rangle$$

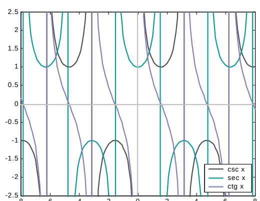
$$y = \angle\sec x = \angle\cos\frac{1}{x} \quad y \in [0, \pi]$$

$$y = \angle\csc x = \angle\operatorname{sen}\frac{1}{x} \quad y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

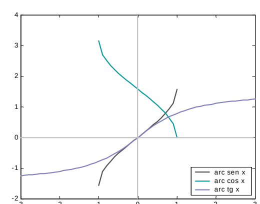
Gráfica 1. Las funciones trigonométricas: $\operatorname{sen} x$, $\cos x$, $\operatorname{tg} x$:



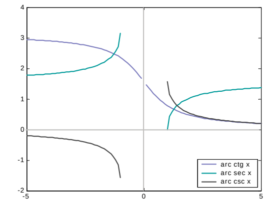
Gráfica 2. Las funciones trigonométricas $\csc x$, $\sec x$, $\operatorname{ctg} x$:



Gráfica 3. Las funciones trigonométricas inversas $\arcsen x$, $\arccos x$, $\operatorname{arctg} x$:



Gráfica 4. Las funciones trigonométricas inversas $\operatorname{arcctg} x$, $\operatorname{arcsec} x$, $\operatorname{arccsc} x$:



IDENTIDADES TRIGONOMÉTRICAS

$$\operatorname{sen}^2\theta + \cos^2\theta = 1$$

$$1 + \operatorname{ctg}^2\theta = \csc^2\theta$$

$$\operatorname{tg}^2\theta + 1 = \sec^2\theta$$

$$\operatorname{sen}(-\theta) = -\operatorname{sen}\theta$$

$$\cos(-\theta) = \cos\theta$$

$$\operatorname{tg}(-\theta) = -\operatorname{tg}\theta$$

$$\operatorname{sen}(\theta + 2\pi) = \operatorname{sen}\theta$$

$$\cos(\theta + 2\pi) = \cos\theta$$

$$\operatorname{tg}(\theta + 2\pi) = \operatorname{tg}\theta$$

$$\operatorname{sen}(\theta + \pi) = -\operatorname{sen}\theta$$

$$\cos(\theta + \pi) = -\cos\theta$$

$$\operatorname{tg}(\theta + \pi) = \operatorname{tg}\theta$$

$$\operatorname{sen}(\theta + n\pi) = (-1)^n \operatorname{sen}\theta$$

$$\cos(\theta + n\pi) = (-1)^n \cos\theta$$

$$\operatorname{tg}(\theta + n\pi) = \operatorname{tg}\theta$$

$$\operatorname{sen}(n\pi) = 0$$

$$\cos(n\pi) = (-1)^n$$

$$\operatorname{tg}(n\pi) = 0$$

$$\operatorname{sen}\left(\frac{2n+1}{2}\pi\right) = (-1)^n$$

$$\cos\left(\frac{2n+1}{2}\pi\right) = 0$$

$$\operatorname{tg}\left(\frac{2n+1}{2}\pi\right) = \infty$$

$$\operatorname{sen}\theta = \cos\left(\theta - \frac{\pi}{2}\right)$$

$$\cos\theta = \operatorname{sen}\left(\theta + \frac{\pi}{2}\right)$$

$$\operatorname{sen}(\alpha \pm \beta) = \operatorname{sen}\alpha\cos\beta \pm \cos\alpha\operatorname{sen}\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \operatorname{sen}\alpha\operatorname{sen}\beta$$

$$\operatorname{tg}(\alpha \pm \beta) = \frac{\operatorname{tg}\alpha \pm \operatorname{tg}\beta}{1 \mp \operatorname{tg}\alpha\operatorname{tg}\beta}$$

$$\operatorname{sen}2\theta = 2\operatorname{sen}\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \operatorname{sen}^2\theta$$

$$\operatorname{tg}2\theta = \frac{2\operatorname{tg}\theta}{1 - \operatorname{tg}^2\theta}$$

$$\operatorname{sen}^2\theta = \frac{1}{2}(1 - \cos 2\theta)$$

$$\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$$

$$\operatorname{tg}^2\theta = \frac{1 - \cos 2\theta}{1 + \cos 2\theta}$$

$$\operatorname{sen}\alpha + \operatorname{sen}\beta = 2\operatorname{sen}\frac{1}{2}(\alpha + \beta)\cdot\cos\frac{1}{2}(\alpha - \beta)$$

$$\operatorname{sen}\alpha - \operatorname{sen}\beta = 2\operatorname{sen}\frac{1}{2}(\alpha - \beta)\cdot\cos\frac{1}{2}(\alpha + \beta)$$

$$\cos\alpha + \cos\beta = 2\cos\frac{1}{2}(\alpha + \beta)\cdot\cos\frac{1}{2}(\alpha - \beta)$$

$$\cos\alpha - \cos\beta = -2\operatorname{sen}\frac{1}{2}(\alpha + \beta)\cdot\operatorname{sen}\frac{1}{2}(\alpha - \beta)$$

$$\operatorname{tg}\alpha \pm \operatorname{tg}\beta = \frac{\operatorname{sen}(\alpha \pm \beta)}{\cos\alpha\cdot\cos\beta}$$

$$\operatorname{sen}\alpha\cdot\cos\beta = \frac{1}{2}\left[\operatorname{sen}(\alpha - \beta) + \operatorname{sen}(\alpha + \beta)\right]$$

$$\operatorname{sen}\alpha\cdot\operatorname{sen}\beta = \frac{1}{2}\left[\cos(\alpha - \beta) - \cos(\alpha + \beta)\right]$$

$$\cos\alpha\cdot\cos\beta = \frac{1}{2}\left[\cos(\alpha - \beta) + \cos(\alpha + \beta)\right]$$

$$\operatorname{tg}\alpha\cdot\operatorname{tg}\beta = \frac{\operatorname{tg}\alpha + \operatorname{tg}\beta}{\operatorname{ctg}\alpha + \operatorname{ctg}\beta}$$

FUNCIONES HIPERBÓLICAS

$$\operatorname{senh} x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\operatorname{tgh} x = \frac{\operatorname{senh} x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\operatorname{ctgh} x = \frac{1}{\operatorname{tgh} x} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

$$\operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}}$$

$$\operatorname{csch} x = \frac{1}{\operatorname{senh} x} = \frac{2}{e^x - e^{-x}}$$

$$\operatorname{senh} : \mathbb{R} \to \mathbb{R}$$

$$\cosh : \mathbb{R} \to [1, \infty\rangle$$

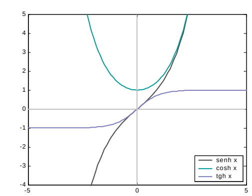
$$\operatorname{tgh} : \mathbb{R} \to \langle -1, 1\rangle$$

$$\operatorname{ctgh} : \mathbb{R} - \{0\} \to \langle -\infty, -1\rangle \cup \langle 1, \infty\rangle$$

$$\operatorname{sech} : \mathbb{R} \to \langle 0, 1]$$

$$\operatorname{csch} : \mathbb{R} - \{0\} \to \mathbb{R} - \{0\}$$

Gráfica 5. Las funciones hiperbólicas $\operatorname{senh} x$, $\cosh x$, $\operatorname{tgh} x$:



FUNCIONES HIPERBÓLICAS INV

$$\operatorname{senh}^{-1} x = \ln\left(x + \sqrt{x^2 + 1}\right), \quad \forall x \in \mathbb{R}$$

$$\cosh^{-1} x = \ln\left(x \pm \sqrt{x^2 - 1}\right), \quad x \ge 1$$

$$\operatorname{tgh}^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right), \quad |x| < 1$$

$$\operatorname{ctgh}^{-1} x = \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right), \quad |x| > 1$$

$$\operatorname{sech}^{-1} x = \ln\left(\frac{1 \pm \sqrt{1 - x^2}}{x}\right), \quad 0 < x \le 1$$

$$\operatorname{csch}^{-1} x = \ln\left(\frac{1}{x} + \frac{\sqrt{x^2 + 1}}{|x|}\right), \quad x \ne 0$$

IDENTIDADES DE FUNCS HIP

$\cosh^2 x - \operatorname{senh}^2 x = 1$

$1 - \operatorname{tgh}^2 x = \operatorname{sech}^2 x$

$\operatorname{ctgh}^2 x - 1 = \operatorname{csch} x$

$\operatorname{senh}(-x) = -\operatorname{senh} x$

$\cosh(-x) = \cosh x$

$\operatorname{tgh}(-x) = -\operatorname{tgh} x$

$\operatorname{senh}(x \pm y) = \operatorname{senh} x \cosh y \pm \cosh x \operatorname{senh} y$

$\cosh(x \pm y) = \cosh x \cosh y \pm \operatorname{senh} x \operatorname{senh} y$

$\operatorname{tgh}(x \pm y) = \dfrac{\operatorname{tgh} x \pm \operatorname{tgh} y}{1 \pm \operatorname{tgh} x \operatorname{tgh} y}$

$\operatorname{senh} 2x = 2 \operatorname{senh} x \cosh x$

$\cosh 2x = \cosh^2 x + \operatorname{senh}^2 x$

$\operatorname{tgh} 2x = \dfrac{2 \operatorname{tgh} x}{1 + \operatorname{tgh}^2 x}$

$\operatorname{senh}^2 x = \dfrac{1}{2}(\cosh 2x - 1)$

$\cosh^2 x = \dfrac{1}{2}(\cosh 2x + 1)$

$\operatorname{tgh}^2 x = \dfrac{\cosh 2x - 1}{\cosh 2x + 1}$

$\operatorname{tgh} x = \dfrac{\operatorname{senh} 2x}{\cosh 2x + 1}$

$e^x = \cosh x + \operatorname{senh} x$

$e^{-x} = \cosh x - \operatorname{senh} x$

OTRAS

$ax^2 + bx + c = 0$

$\Rightarrow\ x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$b^2 - 4ac = \text{discriminante}$

$\exp(\alpha \pm i\beta) = e^{\alpha}(\cos\beta \pm i \operatorname{sen}\beta)$ si $\alpha, \beta \in \mathbb{R}$

LÍMITES

$\lim_{x\to 0}(1+x)^{\frac{1}{x}} = e = 2.71828...$

$\lim_{x\to\infty}\left(1 + \dfrac{1}{x}\right)^x = e$

$\lim_{x\to 0}\dfrac{\operatorname{sen} x}{x} = 1$

$\lim_{x\to 0}\dfrac{1 - \cos x}{x} = 0$

$\lim_{x\to 0}\dfrac{e^x - 1}{x} = 1$

$\lim_{x\to 1}\dfrac{x - 1}{\ln x} = 1$

DERIVADAS

$D_x f(x) = \dfrac{df}{dx} = \lim_{\Delta x\to 0}\dfrac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x\to 0}\dfrac{\Delta y}{\Delta x}$

$\dfrac{d}{dx}(c) = 0$

$\dfrac{d}{dx}(cx) = c$

$\dfrac{d}{dx}(cx^n) = ncx^{n-1}$

$\dfrac{d}{dx}(u \pm v \pm w \pm \cdots) = \dfrac{du}{dx} \pm \dfrac{dv}{dx} \pm \dfrac{dw}{dx} \pm \cdots$

$\dfrac{d}{dx}(cu) = c\dfrac{du}{dx}$

$\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

$\dfrac{d}{dx}(uvw) = uv\dfrac{dw}{dx} + uw\dfrac{dv}{dx} + vw\dfrac{du}{dx}$

$\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v(du/dx) - u(dv/dx)}{v^2}$

$\dfrac{d}{dx}(u^n) = nu^{n-1}\dfrac{du}{dx}$

$\dfrac{dF}{dx} = \dfrac{dF}{du}\cdot\dfrac{du}{dx}$ (Regla de la Cadena)

$\dfrac{du}{dx} = \dfrac{1}{dx/du}$

$\dfrac{dF}{dx} = \dfrac{dF/du}{dx/du}$

$\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{f_2'(t)}{f_1'(t)}$ donde $\begin{cases} x = f_1(t) \\ y = f_2(t) \end{cases}$

DERIVADA DE FUNCS LOG & EXP

$\dfrac{d}{dx}(\ln u) = \dfrac{du/dx}{u} = \dfrac{1}{u}\cdot\dfrac{du}{dx}$

$\dfrac{d}{dx}(\log u) = \dfrac{\log e}{u}\cdot\dfrac{du}{dx}$

$\dfrac{d}{dx}(\log_a u) = \dfrac{\log_a e}{u}\cdot\dfrac{du}{dx}\quad a > 0,\ a \neq 1$

$\dfrac{d}{dx}(e^u) = e^u\cdot\dfrac{du}{dx}$

$\dfrac{d}{dx}(a^u) = a^u \ln a\cdot\dfrac{du}{dx}$

$\dfrac{d}{dx}(u^v) = vu^{v-1}\dfrac{du}{dx} + \ln u\cdot u^v\cdot\dfrac{dv}{dx}$

DERIVADA DE FUNCIONES TRIGO

$\dfrac{d}{dx}(\operatorname{sen} u) = \cos u\dfrac{du}{dx}$

$\dfrac{d}{dx}(\cos u) = -\operatorname{sen} u\dfrac{du}{dx}$

$\dfrac{d}{dx}(\operatorname{tg} u) = \sec^2 u\dfrac{du}{dx}$

$\dfrac{d}{dx}(\operatorname{ctg} u) = -\csc^2 u\dfrac{du}{dx}$

$\dfrac{d}{dx}(\sec u) = \sec u \operatorname{tg} u\dfrac{du}{dx}$

$\dfrac{d}{dx}(\csc u) = -\csc u \operatorname{ctg} u\dfrac{du}{dx}$

$\dfrac{d}{dx}(\operatorname{vers} u) = \operatorname{sen} u\dfrac{du}{dx}$

DERIV DE FUNCS TRIGO INVER

$\dfrac{d}{dx}(\angle\operatorname{sen} u) = \dfrac{1}{\sqrt{1-u^2}}\cdot\dfrac{du}{dx}$

$\dfrac{d}{dx}(\angle\cos u) = -\dfrac{1}{\sqrt{1-u^2}}\cdot\dfrac{du}{dx}$

$\dfrac{d}{dx}(\angle\operatorname{tg} u) = \dfrac{1}{1+u^2}\cdot\dfrac{du}{dx}$

$\dfrac{d}{dx}(\angle\operatorname{ctg} u) = -\dfrac{1}{1+u^2}\cdot\dfrac{du}{dx}$

$\dfrac{d}{dx}(\angle\sec u) = \pm\dfrac{1}{u\sqrt{u^2-1}}\cdot\dfrac{du}{dx}\begin{cases} + \text{ si } u > 1 \\ - \text{ si } u < -1 \end{cases}$

$\dfrac{d}{dx}(\angle\csc u) = \mp\dfrac{1}{u\sqrt{u^2-1}}\cdot\dfrac{du}{dx}\begin{cases} - \text{ si } u > 1 \\ + \text{ si } u < -1 \end{cases}$

$\dfrac{d}{dx}(\angle\operatorname{vers} u) = \dfrac{1}{\sqrt{2u-u^2}}\cdot\dfrac{du}{dx}$

DERIVADA DE FUNCS HIPERBÓLICAS

$\dfrac{d}{dx}\operatorname{senh} u = \cosh u\dfrac{du}{dx}$

$\dfrac{d}{dx}\cosh u = \operatorname{senh} u\dfrac{du}{dx}$

$\dfrac{d}{dx}\operatorname{tgh} u = \operatorname{sech}^2 u\dfrac{du}{dx}$

$\dfrac{d}{dx}\operatorname{ctgh} u = -\operatorname{csch}^2 u\dfrac{du}{dx}$

$\dfrac{d}{dx}\operatorname{sech} u = -\operatorname{sech} u \operatorname{tgh} u\dfrac{du}{dx}$

$\dfrac{d}{dx}\operatorname{csch} u = -\operatorname{csch} u \operatorname{ctgh} u\dfrac{du}{dx}$

DERIVADA DE FUNCS HIP INV

$\dfrac{d}{dx}\operatorname{senh}^{-1} u = \dfrac{1}{\sqrt{1+u^2}}\cdot\dfrac{du}{dx}$

$\dfrac{d}{dx}\cosh^{-1} u = \dfrac{\pm 1}{\sqrt{u^2-1}}\cdot\dfrac{du}{dx},\ u > 1\ \begin{cases} + \text{ si } \cosh^{-1} u > 0 \\ - \text{ si } \cosh^{-1} u < 0 \end{cases}$

$\dfrac{d}{dx}\operatorname{tgh}^{-1} u = \dfrac{1}{1-u^2}\cdot\dfrac{du}{dx},\ |u| < 1$

$\dfrac{d}{dx}\operatorname{ctgh}^{-1} u = \dfrac{1}{1-u^2}\cdot\dfrac{du}{dx},\ |u| > 1$

$\dfrac{d}{dx}\operatorname{sech}^{-1} u = \dfrac{\mp 1}{u\sqrt{1-u^2}}\cdot\dfrac{du}{dx}\begin{cases} - \text{ si } \operatorname{sech}^{-1} u > 0, u \in \langle 0,1\rangle \\ + \text{ si } \operatorname{sech}^{-1} u < 0, u \in \langle 0,1\rangle \end{cases}$

$\dfrac{d}{dx}\operatorname{csch}^{-1} u = -\dfrac{1}{|u|\sqrt{1+u^2}}\cdot\dfrac{du}{dx},\ u \neq 0$

INTEGRALES DEFINIDAS, PROPIEDADES

$\int_a^b \{f(x) \pm g(x)\}dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$

$\int_a^b cf(x)dx = c\cdot\int_a^b f(x)dx \quad c \in \mathbb{R}$

$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

$\int_a^b f(x)dx = -\int_b^a f(x)dx$

$\int_a^a f(x)dx = 0$

$m\cdot(b-a) \leq \int_a^b f(x)dx \leq M\cdot(b-a)$

$\Leftrightarrow m \leq f(x) \leq M \quad \forall x \in [a,b],\ m, M \in \mathbb{R}$

$\int_a^b f(x)dx \leq \int_a^b g(x)dx$

$\Leftrightarrow f(x) \leq g(x) \quad \forall x \in [a,b]$

$\left|\int_a^b f(x)dx\right| \leq \int_a^b |f(x)|dx$ si $a < b$

INTEGRALES

$\int a\,dx = ax$

$\int af(x)dx = a\int f(x)dx$

$\int (u \pm v \pm w \pm \cdots)dx = \int u\,dx \pm \int v\,dx \pm \int w\,dx \pm \cdots$

$\int u\,dv = uv - \int v\,du$ (Integración por partes)

$\int u^n du = \dfrac{u^{n+1}}{n+1} \quad n \neq -1$

$\int \dfrac{du}{u} = \ln|u|$

INTEGRALES DE FUNCS LOG & EXP

$\int e^u du = e^u$

$\int a^u du = \dfrac{a^u}{\ln a}\begin{cases} a > 0 \\ a \neq 1 \end{cases}$

$\int ua^u du = \dfrac{a^u}{\ln a}\cdot\left(u - \dfrac{1}{\ln a}\right)$

$\int ue^u du = e^u(u-1)$

$\int \ln u\,du = u\ln u - u = u(\ln u - 1)$

$\int \log_a u\,du = \dfrac{1}{\ln a}(u\ln u - u) = \dfrac{u}{\ln a}(\ln u - 1)$

$\int u\log_a u\,du = \dfrac{u^2}{4}\cdot(2\log_a u - 1)$

$\int u\ln u\,du = \dfrac{u^2}{4}(2\ln u - 1)$

INTEGRALES DE FUNCS TRIGO

$\int \operatorname{sen} u\,du = -\cos u$

$\int \cos u\,du = \operatorname{sen} u$

$\int \sec^2 u\,du = \operatorname{tg} u$

$\int \csc^2 u\,du = -\operatorname{ctg} u$

$\int \sec u \operatorname{tg} u\,du = \sec u$

$\int \csc u \operatorname{ctg} u\,du = -\csc u$

$\int \operatorname{tg} u\,du = -\ln|\cos u| = \ln|\sec u|$

$\int \operatorname{ctg} u\,du = \ln|\operatorname{sen} u|$

$\int \sec u\,du = \ln|\sec u + \operatorname{tg} u|$

$\int \csc u\,du = \ln|\csc u - \operatorname{ctg} u|$

$\int \operatorname{sen}^2 u\,du = \dfrac{u}{2} - \dfrac{1}{4}\operatorname{sen} 2u$

$\int \cos^2 u\,du = \dfrac{u}{2} + \dfrac{1}{4}\operatorname{sen} 2u$

$\int \operatorname{tg}^2 u\,du = \operatorname{tg} u - u$

$\int \operatorname{ctg}^2 u\,du = -(\operatorname{ctg} u + u)$

$\int u \operatorname{sen} u\,du = \operatorname{sen} u - u\cos u$

$\int u\cos u\,du = \cos u + u\operatorname{sen} u$

INTEGRALES DE FUNCS TRIGO INV

$\int \angle\operatorname{sen} u\,du = u\angle\operatorname{sen} u + \sqrt{1-u^2}$

$\int \angle\cos u\,du = u\angle\cos u - \sqrt{1-u^2}$

$\int \angle\operatorname{tg} u\,du = u\angle\operatorname{tg} u - \ln\sqrt{1+u^2}$

$\int \angle\operatorname{ctg} u\,du = u\angle\operatorname{ctg} u + \ln\sqrt{1+u^2}$

$\int \angle\sec u\,du = u\angle\sec u - \ln\left(u + \sqrt{u^2-1}\right)$

$= u\angle\sec u - \angle\cosh u$

$\int \angle\csc u\,du = u\angle\csc u + \ln\left(u + \sqrt{u^2-1}\right)$

$= u\angle\csc u + \angle\cosh u$

INTEGRALES DE FUNCS HIP

$\int \operatorname{senh} u\,du = \cosh u$

$\int \cosh u\,du = \operatorname{senh} u$

$\int \operatorname{sech}^2 u\,du = \operatorname{tgh} u$

$\int \operatorname{csch}^2 u\,du = -\operatorname{ctgh} u$

$\int \operatorname{sech} u \operatorname{tgh} u\,du = -\operatorname{sech} u$

$\int \operatorname{csch} u \operatorname{ctgh} u\,du = -\operatorname{csch} u$

$\int \operatorname{tgh} u\,du = \ln\cosh u$

$\int \operatorname{ctgh} u\,du = \ln|\operatorname{senh} u|$

$\int \operatorname{sech} u\,du = \angle\operatorname{tg}(\operatorname{senh} u)$

$\int \operatorname{csch} u\,du = -\operatorname{ctgh}^{-1}(\cosh u)$

$= \ln\operatorname{tgh}\dfrac{1}{2}u$

INTEGRALES DE FRAC

$\int \dfrac{du}{u^2+a^2} = \dfrac{1}{a}\angle\operatorname{tg}\dfrac{u}{a}$

$= -\dfrac{1}{a}\angle\operatorname{ctg}\dfrac{u}{a}$

$\int \dfrac{du}{u^2-a^2} = \dfrac{1}{2a}\ln\dfrac{u-a}{u+a} \quad (u^2 > a^2)$

$\int \dfrac{du}{a^2-u^2} = \dfrac{1}{2a}\ln\dfrac{a+u}{a-u} \quad (u^2 < a^2)$

INTEGRALES CON $\sqrt{\ }$

$\int \dfrac{du}{\sqrt{a^2-u^2}} = \angle\operatorname{sen}\dfrac{u}{a}$

$= -\angle\cos\dfrac{u}{a}$

$\int \dfrac{du}{\sqrt{u^2 \pm a^2}} = \ln\left(u + \sqrt{u^2 \pm a^2}\right)$

$\int \dfrac{du}{u\sqrt{a^2 \pm u^2}} = \dfrac{1}{a}\ln\left|\dfrac{u}{a + \sqrt{a^2 \pm u^2}}\right|$

$\int \dfrac{du}{u\sqrt{u^2-a^2}} = \dfrac{1}{a}\angle\cos\dfrac{a}{u}$

$= \dfrac{1}{a}\angle\sec\dfrac{u}{a}$

$\int \sqrt{a^2-u^2}\,du = \dfrac{u}{2}\sqrt{a^2-u^2} + \dfrac{a^2}{2}\angle\operatorname{sen}\dfrac{u}{a}$

$\int \sqrt{u^2 \pm a^2}\,du = \dfrac{u}{2}\sqrt{u^2 \pm a^2} \pm \dfrac{a^2}{2}\ln\left(u + \sqrt{u^2 \pm a^2}\right)$

MÁS INTEGRALES

$\int e^{au}\operatorname{sen} bu\,du = \dfrac{e^{au}(a\operatorname{sen} bu - b\cos bu)}{a^2+b^2}$

$\int e^{au}\cos bu\,du = \dfrac{e^{au}(a\cos bu + b\operatorname{sen} bu)}{a^2+b^2}$

ALGUNAS SERIES

$f(x) = f(x_0) + f'(x_0)(x-x_0) + \dfrac{f''(x_0)(x-x_0)^2}{2!}$

$+ \cdots + \dfrac{f^{(n)}(x_0)(x-x_0)^n}{n!}$: Taylor

$f(x) = f(0) + f'(0)x + \dfrac{f''(0)x^2}{2!}$

$+ \cdots + \dfrac{f^{(n)}(0)x^n}{n!}$: Maclaurin

$e^x = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \cdots + \dfrac{x^n}{n!} + \cdots$

$\operatorname{sen} x = x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \dfrac{x^7}{7!} + \cdots + (-1)^{n-1}\dfrac{x^{2n-1}}{(2n-1)!}$

$\cos x = 1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} - \dfrac{x^6}{6!} + \cdots + (-1)^{n-1}\dfrac{x^{2n-2}}{(2n-2)!}$

$\ln(1+x) = x - \dfrac{x^2}{2} + \dfrac{x^3}{3} - \dfrac{x^4}{4} + \cdots + (-1)^{n-1}\dfrac{x^n}{n}$

$\angle\operatorname{tg} x = x - \dfrac{x^3}{3} + \dfrac{x^5}{5} - \dfrac{x^7}{7} + \cdots + (-1)^{n-1}\dfrac{x^{2n-1}}{2n-1}$

ÁLGEBRA LINEAL

Def. *El **determinante** de una matriz*

$$A=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

está dado por

$$\det A=\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}=a_{11}a_{22}-a_{12}a_{21} \quad .$$

Def. *El **determinante** de una matriz*

$$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

está dado por

$$\det A=\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}=\begin{matrix} a_{11}\cdot a_{22}\cdot a_{33}+a_{12}\cdot a_{23}\cdot a_{31} \\ +a_{13}\cdot a_{21}\cdot a_{32}-a_{11}\cdot a_{23}\cdot a_{32} \\ -a_{12}\cdot a_{21}\cdot a_{33}-a_{13}\cdot a_{22}\cdot a_{31} \end{matrix} \quad .$$